hep-ph/0312237

SHEP/0334

Canonical normalisation and Yukawa matrices

S. F. King and I. N. R. Peddie

School of Physics and Astronomy, University of Southampton

Southampton, SO17 1BJ, U.K.

Abstract

We highlight the important rôle that canonical normalisation of kinetic terms in flavour models based on family symmetries can play in determining the Yukawa matrices. Even though the kinetic terms may be correctly canonically normalised to begin with, they will inevitably be driven into a non-canonical form by a similar operator expansion to that which determines the Yukawa operators. Therefore in models based on family symmetry canonical re-normalisation is mandatory before the physical Yukawa matrices can be extracted. *In nearly all examples in the literature this is not done.* As an example we perform an explicit calculation of such mixing associated with canonical normalisation of the Kähler metric in a supersymmetric model based on $SU(3)$ family symmetry, where we show that such effects can significantly change the form of the Yukawa matrix. In principle quark mixing could originate entirely from canonical normalisation, with only diagonal Yukawa couplings before canonical normalisation.

February 1, 2008

1 Introduction

There is great interest in the literature in trying to understand the hierarchical pattern of Standard Model fermion masses, the smallness of the quark mixing angles and the two large and one small neutrino mixing angles. One popular way of doing this is to extend either the Standard Model, or one of its more common supersymmetric extensions, by adding a gauge or global family symmetry, G_F which is subsequently broken [1].

In such models based on family symmetry G_F, Yukawa couplings arise from Yukawa operators which are typically non-renormalisable and involve extra heavy scalar fields, ϕ, coupling to the usual three fields, for example:

$$\mathcal{O}_Y = F\overline{F}H\left(\frac{\phi}{M}\right)^n \tag{1}$$

where F represents left-handed fermion fields, $\overline{F}$ represents the CP-conjugate of right-handed fermion fields, H represents the Higgs field, and M is a heavy mass scale which acts as an ultraviolet (UV) cutoff. In the context of supersymmetric (SUSY) field theories, all the fields become superfields. The operators in Eq.1 are invariant under G_F, but when the scalar fields ϕ develop vacuum expectation values (vevs) the family symmetry is thereby broken and the Yukawa couplings are generated. The resulting Yukawa couplings are therefore effective couplings expressed in terms of an expansion parameter, ϵ, which is the ratio of the vev of the heavy scalar field to the UV cut-off, $\epsilon = \frac{\langle\phi\rangle}{M}$. Explaining the hierarchical form of the Yukawa matrices then reduces to finding an appropriate symmetry G_F and field content which leads to acceptable forms of Yukawa matrices, and hence fermion masses and mixing angles, at the high energy scale.

Over recent years there has been a huge activity in this family symmetry and operator approach to understanding the fermion masses and mixing angles [2], including

neutrino masses and mixing angles [3]. However, as we shall show in this paper, in analysing such models it is important to also consider the corresponding operator expansion of the kinetic terms. The point is that, even though the kinetic terms may be correctly canonically normalised to begin with, they will inevitably be driven to a non-canonical form by a similar operator expansion to that which determines the Yukawa operator. In order to extract reliable predictions of Yukawa matrices, it is mandatory to canonically re-normalise the kinetic terms once again before proceeding. *In nearly all examples in the literature this is not done.* The main point of our paper is thus to highlight this effect and to argue that it is sufficiently important that it must be taken into account before reliable predictions can be obtained.

Many approaches combine the family symmetry and operator approach with supersymmetric grand unified theories (SUSY GUTs) [2, 3]. Such models tend to be more constraining, because the Yukawa matrices at the high scale should have the same form, up to small corrections from the breaking of the unified symmetry. The same comments we made above also apply in the framework of SUSY GUTs. In the SUSY case the Yukawa operators arise from the superpotential W, and the kinetic terms and scalar masses, as well as gauge interaction terms come from the Kähler potential, K. In nearly all examples in the literature the superpotential W has been analysed independently of the Kähler potential, K, leading to most of the published results being inconsistent. The correct procedure which should be followed is as follows.

To be consistent, the Kähler potential, K, should also be written down to the same order M^{-n} as the superpotential W. Having done this, one should proceed to calculate the elements of the Kähler metric, $\tilde{K}_{ij}$, which are second derivatives with respect to fields of the Kähler potential $\tilde{K}_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi_i \partial \phi_j^\dagger}$. However, in order to have canonically normalised kinetic terms, the Kähler metric has to itself be canonically normalised $\tilde{K}_{i\bar{j}} = \delta_{i\bar{j}}$. In making this transformation, the superfields in the Kähler potential are

first being mixed and then rescaled. Once this has been done, the superfields in the superpotential must be replaced by the canonically normalised fields.

Canonical normalisation is not of course a new invention, it has been known since the early days of supergravity [5]. However, as we have mentioned, for some reason this effect has been largely ignored in the model building community. A notable exception is the observation some time ago by Dudas, Pokorski and Savoy [6], that the act of canonical normalisation will change the Yukawa couplings, and could serve to cover up “texture zeros”, which are due to an Abelian family symmetry which does not allow a specific entry in the Yukawa matrix and is therefore manifested as a zero at high energies. This issue has been resurrected for abelian family models recently [7]. However, as we have already noted, this observation has not been pursued or developed in the literature, but instead has been largely ignored.

In this paper we consider the issue of canonical normalisation in the framework of non-Abelian symmetries, in which the Yukawa matrices are approximately symmetric. In such a framework we show that the effects of canonical normalisation extend beyond the filling in of “texture zeros”, and can also change the expansion order of the leading non-zero entries in the Yukawa matrix. As an example we perform an explicit calculation of such mixing associated with canonical normalisation of the Kähler metric in a recent supersymmetric model based on $SU(3)$ family symmetry where we show that such effects can significantly change the form of the Yukawa matrix. The $SU(3)$ model we consider is a grossly simplified version of the realistic model in [4], where we only consider the case of a single expansion parameter and perform our calculations in the 23 sector of the theory for simplicity, although we indicate how the results can straightforwardly be extended to the entire theory. An alternative scenario in which in principle quark mixing could originate entirely from canonical normalisation, with only diagonal Yukawa couplings before canonical normalisation, is also discussed.

The outline of the rest of the paper is as follows. In section 2 we discuss the issues surrounding canonical normalisation in the Standard Model supplemented by a family symmetry, first without then with SUSY. In the SUSY case we discuss the scalar mass squared and Yukawa matrices for two types of Kähler potentia where only one superfield contributes to supersymmetry breaking. In section 3 we discuss a particular model in some detail as a concrete example, namely the simplified $SU(3)$ family symmetry model, focusing on the second and third generations of matter, later indicating how the results can be extended to all three families. We conclude in section 5.

2 Canonical normalisation

2.1 Standard Model with a Family Symmetry

In this section we first consider extending the Standard Model gauge group with a family symmetry, under which each generation has a different charge (for abelian family symmetries) or representation (for non-abelian family symmetries). The family symmetry typically prohibits renormalisable Yukawa couplings (except possibly for the third family) but allows non-renormalisable operators, for example:

$$\mathcal{O}_Y = F^i H \overline{F}^j \frac{\phi_i \phi_j}{M^2} \tag{2}$$

where i, j are generation indices, M is some appropriate UV cutoff, F represents left-handed fermion fields, and $\overline{F}$ represents CP-conjugates of right-handed fermion fields, and H is a Higgs field. When the flavon scalar field ϕ gets a vev, which breaks the family symmetry, effective Yukawa couplings are generated:

$$Y_{ij} = \frac{\langle \phi_i \rangle \langle \phi_j \rangle}{M^2} \tag{3}$$

The effective Yukawa matrices are determined by the operators allowed by the symmetries of the model, $G_F \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, as well as the form that the

vev of ϕ takes.

Even though the kinetic terms are correctly canonically normalised to begin with, they will receive non-renormalisable corrections arising from operators allowed by the family symmetry, which will cast them into non-canonical form. For example,

$$F_i^\dagger \not{\partial} F^j \left(\delta_j^i + \frac{\phi^i \phi_j^\dagger}{M^2} \right) \tag{4}$$

This leads to a non-canonical kinetic term when ϕ is replaced by its vev. It is therefore mandatory to perform a further canonical re-normalisation of the kinetic terms, before analysing the physical Yukawa couplings. The canonical normalisation amounts to a transformation which is not unitary but which gives all the fields canonical kinetic terms. The kinetic part of a theory with a Higgs scalar field H, a fermionic field F^i and the field strength tensor $F^{\mu\nu}$ corresponding to a gauge field A^μ when canonical will look like:

$$\mathcal{L}_{\mathrm{canonical}} = \partial_\mu H \partial^\mu H^* + F_i^\dagger \not{\partial} F^i - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{5}$$

Once we have done this normalisation, we have to rewrite all of our interactions in terms of the canonical fields with the shifted fields.

The important point we wish to emphasise is that all the interaction terms should be expressed in terms of canonical fields, before making any physical interpretation. If this is not done, as is often the case in the literature, then the results will not be reliable.

2.2 SUSY Standard Model with Family Symmetry

In the context of supersymmetric theories, it turns out to be possible to automatically canonically normalise all the fields in the theory at once. However these transformations are not always simple, and in practice calculating the relevant transformations may well turn out to be intractable for any given model.

The aim of SUSY model builders with respect to flavour is two-fold. The primary wish is to generate a set of effective Yukawa matrices which successfully predict the quark and lepton masses and mixing angles as measured by experiment. However, because of the parameters associated with softly broken SUSY models, there exist dangerous one-loop diagrams which lead to processes such as $b \to s\gamma$ and $\mu \to e\gamma$ at rates much greater than predicted by the Standard Model and also much greater than measured by experiment. A successful SUSY theory of flavour will therefore successfully describe fermion masses and mixing angles, while simultaneously controlling such flavour changing processes induced by loop diagrams involving sfermion masses which are off-diagonal in the basis where the quarks and leptons are diagonal.

In a SUSY or supergravity (SUGRA) model, very often the starting point in addressing the flavour problem is to propose a set of symmetries that will give rise to non-renormalisable superpotential operators which will lead to a hierarchical form for our Yukawa matrices, arising from some effective Yukawa operators as discussed previously. Extra fields, ϕ are introduced that spontaneously break the extra family symmetries. The general form of the superpotential is :

$$W = F^i \overline{F}^j H w_{ij} \left(\phi / M \right) \tag{6}$$

Here $w_{ij}(\phi/M)$ is a general function of the extra fields, ϕ, which has mass dimension zero and contracts with $F^i\overline{F}^j$ to make W a singlet of the extended symmetry group.

In models of this type, the amount of flavour violation is proportional to the size of the off-diagonal elements in the scalar mass matrices at the electroweak (EW) scale when the scalar mass matrices have been rotated to the basis where the Yukawas are diagonal (the super-CKM basis). Since the quark mixing angles are small, this suggests that any large scalar mixings at the electroweak scale would remain large when in the super-CKM basis. Since we would generally not expect the RG running of the scalar mass matrices to tune large off-diagonal values to zero, we would expect

to be in trouble if there are large off diagonal scalar mass mixings predicted at the high energy scale. This scale might be, for example, the unification scale in a SUSY GUT.

We now proceed to outline the argument that this will not be a problem in general terms in the simplest case, gravity-mediated supersymmetry breaking, where the breaking is due to a single hidden sector superfield, S. As examples, we shall consider the Kähler potential in two forms. The first form is:

$$K_1 = \ln(S + \overline{S}) + F^i F^\dagger_j k^j_i\,(\phi/M) + \overline{F}^i \overline{F}^\dagger_j \overline{k}^j_i\,(\phi/M) \tag{7}$$

The second form we consider is:

$$K_2 = \frac{S\overline{S}}{M^2}\left(F^i F^\dagger_j k^j_i\,(\phi/M) + \overline{F}^i \overline{F}^\dagger_j \overline{k}^j_i\,(\phi/M)\right) \tag{8}$$

Here $k(\phi)$ and $\overline{k}(\phi)$ represent functions of the various ϕ fields that can be contracted with the matter fields to make the Kähler potential a singlet and of the correct mass dimension.

Since we are looking at gravity-mediated SUSY breaking, we may use the SUGRA equations which relate the *non-canonically normalised* soft scalar mass squared matrices $m^2_{\overline{a}b}$ in the soft SUSY breaking Lagrangian to the Kähler metric $\tilde{K}_{\overline{a}b} = \frac{\partial^2 K}{\partial\phi^\dagger_a \partial\phi^b}$, and the vevs of the auxiliary fields which are associated with the supersymmetry breaking, F_m [5]:

$$m^2_{\overline{a}b} = m^2_{3/2}\tilde{K}_{\overline{a}b} - F_{\overline{m}}\left(\partial_{\overline{m}}\partial_n \tilde{K}_{\overline{a}b} - \partial_{\overline{m}}\tilde{K}_{\overline{a}c}(\tilde{K}^{-1})_{c\overline{d}}\partial_n \tilde{K}_{\overline{d}b}\right) F_n \tag{9}$$

where we have assumed a negligibly small cosmological constant. Roman indices from the middle of the alphabet are taken to be over the hidden sector fields, which in our case can only be the singlet field S associated with SUSY breakdown. As it happens, for both K_1 and K_2, the *non-canonically normalised* mass matrix reduces to:

$$m^2_{\overline{a}b} = m^2_{3/2}\tilde{K}_{\overline{a}b} \tag{10}$$

This is obvious for K_1, since the Kähler metric doesn't involve S, so partial derivatives with respect to S will give zero. To see that eq. (9) reduces to eq. (10) for K_2, is less obvious. We first write:

$$\tilde{K}_{\overline{a}b} = \frac{S\overline{S}}{M^2}\mathcal{M}_{\overline{a}b} \tag{11}$$

Substituting this into eq. (9) gives a non-canonically normalised scalar mass squared matrix:

$$m^2_{\overline{a}b} = m^2_{3/2}\tilde{K}_{\overline{a}b} - F_{\overline{S}}\left(\frac{1}{M^2}\mathcal{M} - \frac{S}{M^2}\mathcal{M}\frac{M^2}{S\overline{S}}\mathcal{M}^{-1}\mathcal{M}\frac{\overline{S}}{M^2}\right)F_S \tag{12}$$

It is clear that eq. (12) reduces to eq. (10). However, the physical states are those for which the Kähler metric is canonically normalised, $\tilde{K} = 1$. This is attained by $\tilde{P}^\dagger\tilde{K}\tilde{P} = 1$. In order to canonically normalise the mass matrix, we apply the same transformation, and find that the *canonically normalised* squark mass squared matrix then takes the universal form:

$$m^2_{\text{c.n.}} = m^2_{3/2}\mathbf{1} \tag{13}$$

We conclude that models with Kähler potentials like K_1 or K_2 will result in universal sfermion masses at the high-energy scale. Of course all this is well known, and it has long been appreciated that this would tame the second part of the flavour problem, flavour violating decays. However, what is less well appreciated at least amongst the model building community, is that canonical normalisation corresponds to redefining the fields in the Kähler potential, and one must therefore also redefine these fields in the same way in the superpotential. Unless this is done consistently it could lead to a problem with the first part of the flavour problem, because the shifted fields may well no longer lead to a phenomenologically successful prediction of the masses and mixing angles for the quarks and leptons.

The procedure is easily formalised in SUSY theories by writing all the fields as ψ_i, $i = 1 \cdots n$. Then, at least in principle, if not in practice, a matrix, P can be found which transforms the vector ψ to the vector ψ_c for which the Kähler metric is

canonically normalised:

$$\psi \to \psi_c = P \cdot \psi \tag{14}$$

This procedure can be followed providing ψ if P is not a singular matrix. Having found P, one we can write $\psi = P^{-1}\psi_c$. We can then substitute these into the superpotential now expressed in terms of fields which correspond to a correctly canonically normalised Kähler metric:

$$W' = F'^{i}\overline{F}'^{j} H' w'_{ij} \left(\phi'/M\right) \tag{15}$$

It should be noted that despite the shifts originating from the Kähler potential, which is a non-holomorphic function of the fields, the shifted fields only depend on the vevs of the flavon and its hermitian conjugate. Therefore, the shifted superpotential will remain holomorphic in terms of fields. That is to say, the shifted fields will be a function of the corresponding unshifted field, and the vevs which break the family symmetry. Here θ represents any field, $\langle\phi\rangle$ the vev of a flavon field, and $\left\langle\phi^\dagger\right\rangle$ the vev of the hermitian conjugate of a flavon field:

$$\theta \to \theta'\left(\theta, \langle\phi\rangle, \left\langle\phi^\dagger\right\rangle\right) \tag{16}$$

At this point, if we had a specific model, we would then need to check that the Yukawas are viable. This is then the correct procedure which must be followed in analysing a general model. We now turn to a particular example which illustrates the effects described above, in the framework on a non-Abelian family symmetry.

3 A SUSY Model based on $SU(3)$ family symmetry

3.1 The quark sector

As an example of the general considerations above, and in order to determine the quantitative effects of canonical normalisation, we now turn to a particular example

based on SU(3)$_F$ family symmetry. As mentioned the model we consider is a simplified version of the realistic model by King and Ross [4] in which we assume only a single expansion parameter. For simplicity, we shall also ignore the presence of the first generation, although later we shall indicate how the results may be extended to the three family case. This model is based on a SUSY Pati-Salam model with a gauged SU(3) family symmetry extended by a $Z_2 \otimes U(1)$ global symmetry. As shown in Table 1, the left-handed matter is contained in F^i, the right-handed matter is contained in a left-handed field $\overline{F}^i$. The MSSM Higgs doublets are contained in H; Σ is a field which has broken $SO(10)$ to $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$. There are two $SU(3)_F$-breaking fields, ϕ_3 and ϕ_{23}.

Field	$SU(3)_F$	$SU(4)_{PS}$	$SU(2)_L$	$SU(2)_R$	Z_2	$U(1)$
F	3	4	2	1	+	0
$\overline{F}$	3	$\overline{4}$	1	2	+	0
H	1	1	2	2	+	8
Σ	1	15	1	1	+	2
ϕ_3	$\overline{3}$	1	1	1	-	-4
ϕ_{23}	$\overline{3}$	1	1	1	+	-5

Table 1: The field content of the toy model

The superpotential has to be a singlet under the combined gauge group $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_F$ and also neutral under $Z_2 \otimes U(1)$. Because of this, the standard Yukawa superpotential:

$$W = F^i \overline{F}^j H Y_{ij} \tag{17}$$

is not allowed because of the $Z_2 \otimes U(1)$. As such, we have to move to a superpotential containing non-renormalisable terms. We view this as being the superpotential corresponding to a supersymmetric effective field theory, where some heavy messenger states and their superpartners have been integrated out. Then, assuming that the messenger states have the same approximate mass scale, we write:

$$W = \frac{1}{M^2} a_1 F^i \overline{F}^j H \phi_{3,i} \phi_{3,j} + \frac{1}{M^3} a_2 F^i \overline{F}^j H \Sigma \phi_{23,i} \phi_{23,j} \tag{18}$$

The a_i are parameters that are expected to be of the order of unity, M is the appropriate UV cutoff of the effective field theory. This will clearly lead to a set of effective Yukawa terms when the fields ϕ_3 and ϕ_{23} gain vevs which break the family symmetry.

We choose the vacuum structure after King and Ross [4]:

$$\langle\phi_3\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} a \; ; \; \langle\phi_{23}\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} b \; ; \; \langle\Sigma\rangle = \sigma \tag{19}$$

And we then trade these for a single expansion parameter, $\epsilon \approx \frac{1}{10}$.

$$\frac{a}{M} = \sqrt{\epsilon} \; ; \; \frac{b}{M} = \epsilon \; ; \; \frac{\sigma}{M} = \epsilon \tag{20}$$

Substituting eqs. (19, 20) into eq. (18), we can write down our high-energy Yukawa matrix:

$$Y_{\text{n.c.}} = \begin{pmatrix} a_2\epsilon^3 & a_2\epsilon^3 \\ a_2\epsilon^3 & a_1\epsilon + a_2\epsilon^3 \end{pmatrix} \tag{21}$$

We write it as $Y_{\text{n.c.}}$ to represent the fact that it is the Yukawa matrix corresponding to the non-canonical Kähler metric.

3.2 The squark sector

In order to write down the squark mass matrices, the first step is to write down our Kähler potential. This should be the most general Kähler potential consistent with the symmetries of our model up to the same order in inverse powers of the UV cutoff as the superpotential is taken to. In our case, this is M^{-3}. However, from the general arguments of section 2, we know that if we pick our Kähler potential, K to be of the form as K_1 (eq. (7)) or K_2 (eq. (8)) then we will have universal scalars.

The non-canonical form of the scalar mass-squared matrix is:

$$m^2_{\text{n.c.}} \sim \begin{pmatrix} 1+\epsilon & \epsilon^2 \\ \epsilon^2 & 1+\epsilon \end{pmatrix} \tag{22}$$

However, we already know exactly what the canonical form of this matrix will look like:

$$m^2 = m^2_{3/2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{23}$$

This universal form is a direct result of the simple supersymmetry breaking mechanism that we have and canonical normalisation, and is independent of other details about the model.

3.3 Kähler potential for the model

We saw in the previous subsection that we will not end up with dangerous off-diagonal elements in the scalar mass matrices for general Kähler potentials of the type we are going to look at. We must now write down our Kähler potential. We choose this to be of the same form K_1. There will be no M^{-3} terms, so it will suffice to write this down up to $\mathcal{O}(M^{-2})$.

The matrices which diagonalise the Kähler metric will in general be large and intractable. In order to proceed, we will have to make some simplifying assumptions. We first assume that the Kähler metric $\tilde{K}_{\overline{a}b} = \frac{\partial^2 K}{\partial \phi_a^\dagger \partial \phi^b}$ is block diagonal, of the form:

$$\tilde{K}_{\overline{a}b} = \begin{pmatrix} \tilde{K}_{LH} & & & & \\ & \tilde{K}_{RH} & & & \\ & & \tilde{K}_\phi & & \\ & & & \tilde{K}_\Sigma & \\ & & & & \tilde{K}_H \end{pmatrix}_{\overline{a}b} \tag{24}$$

In this, $\tilde{K}_{LH}$ represents the block for chiral superfields, F, containing left-handed matter; $\tilde{K}_{RH}$ represents chiral superfields, $\overline{F}$, containing right-handed matter; $\tilde{K}_\phi$ represents the $SU(3)_F$ breaking Higgs fields, ϕ_{23} and ϕ_3; $\tilde{K}_\Sigma$ represents the block for the Higgs field that break the GUT symmetry down to the MSSM gauge group, Σ; finally, the block $\tilde{K}_H$ represents the block corresponding to the MSSM Higgs fields, H. The block diagonal assumption is equivalent to switching off some terms in the Kähler potential. The remaining terms in the Kähler potential are listed below:

$$\begin{aligned} K \quad &= \ln(S + \overline{S}) + b_0 F^i F_i^\dagger + \frac{1}{M^2} F^i F_j^\dagger \left\{ \phi_3^k \phi_{3,l}^\dagger (b_1 \delta_i^l \delta_k^j + b_2 \delta_i^j \delta_k^l) \right. \\ &\left. + \phi_{23}^k \phi_{23,l}^\dagger (b_3 \delta_i^l \delta_k^j + b_4 \delta_i^j \delta_k^l) + b_5 H H^\dagger \delta_i^j + b_6 \Sigma \Sigma^\dagger \delta_i^j \right\} \end{aligned}$$

$$
\begin{aligned}
&+c_0\overline{F}^i\overline{F}^\dagger_i+\frac{1}{M^2}\overline{F}^i\overline{F}^\dagger_j\left\{\phi_3^k\phi^\dagger_{3,l}(c_1\delta^l_i\delta^j_k+c_2\delta^j_i\delta^l_k)\right.\\
&\left.+\phi_{23}^k\phi^\dagger_{23,l}(c_3\delta^l_k\delta^j_k+c_4\delta^j_i\delta^l_k)+c_5HH^\dagger\delta^j_i+c_6\Sigma\Sigma^\dagger\delta^j_i\right\}\\
&+d_1\phi_3^i\phi^\dagger_{3,i}+d_2\phi_{23}^i\phi^\dagger_{23,i}+d_3HH^\dagger+d_4\Sigma\Sigma^\dagger\\
&+\frac{1}{M^2}\left\{\phi_3^i\phi^\dagger_{3,j}\phi_3^k\phi^\dagger_{3,l}d_5\delta^j_i\delta^l_k+\phi_3^i\phi^\dagger_{3,j}\phi_{23}^k\phi^\dagger_{23,l}(d_6\delta^j_i\delta^l_k+d_7\delta^j_k\delta^l_i)\right.\\
&\left.+\phi_{23}^i\phi^\dagger_{23,j}\phi_{23}^k\phi^\dagger_{23,l}d_8\delta^j_i\delta^l_k+d_9HH^\dagger HH^\dagger+d_{10}\Sigma\Sigma^\dagger\Sigma\Sigma^\dagger\right\} \qquad (25)
\end{aligned}
$$

Having done this, we now need to calculate the Kähler metric $\tilde{K}$. But since we have set K up specifically such that it is block diagonal, we can instead work out the non-zero blocks, $\tilde{K}_{LH}$, $\tilde{K}_{RH}$, $\tilde{K}_\phi$, $\tilde{K}_\Sigma$ and $\tilde{K}_H$. Once we have done so, we need to canonically normalise them. This is done in two stages. The first is a unitary transformation to diagonalise each block $\tilde{K}_i$:

$$\mathcal{L}\supset F^\dagger\tilde{K}F\rightarrow(F^\dagger U)(U^\dagger\tilde{K}U)(U^\dagger F)=F'\tilde{K}'F^{\dagger\prime} \qquad (26)$$

The mixed Kähler metric, $\tilde{K}'$, is now diagonal. Then we rescale the fields by a diagonal matrix R such that $R_i=(\tilde{K}'_i)^{-1/2}$. These new superfields are then canonically normalised.

Then:

$$\mathcal{L}\supset(F^\dagger UR^{-1})\underbrace{(RU^\dagger\tilde{K}UR)}_{\mathbf{1}}(R^{-1}U^\dagger F) \qquad (27)$$

If we call P the matrix which converts F to the canonical field F_c, then we can note two things. Firstly $P=R^{-1}U^\dagger$. Secondly, we can read off:

$$F^\dagger P^\dagger PF=F^\dagger UR^{-1}R^{-1}U^\dagger F=F^\dagger\tilde{K}F \qquad (28)$$

So the Kähler metric is equal to $P^\dagger P$.

The important point to note is that in canoncally normalising, we have redefined our superfields, so we must also redefine them in our superpotential. This is discussed in the next section.

3.4 Yukawa sector after canonical normalisation

In this section we return to the important question of the form of the Yukawa matrices in the correct canonically normalised basis. In order to do this we would have to calculate the shifting in all of the fields in the superpotential. Unfortunately, algebraically diagonalising the sub-block $\tilde{K}_\phi$ is intractable, even for such a simple model. We therefore make a second assumption and neglect the effects of canonical normalisation arising from this sector, although we shall correctly consider the effects of canonical normalisation arising from all the other sectors.

Even making this assumption, the expressions we get are not especially pleasant. We then substitute in the form of the vevs (eq. (19) and eq. (20)). Having done this, we then expand the cofactors of $F^i\overline{F}^j H$ as a power series in ϵ around the point $\epsilon = 0$. The cofactors of ϵ^n are quite complicated, so we only write out here the expression for the effective Yukawa for the 22 element. The full expressions for all four elements are listed in Appendix A.

$$Y_{23} = -a_1 \frac{b_3}{\sqrt{b_0 c_0} b_1}\epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}}\epsilon^3 + a_1 \frac{b_3(b_2 c_0 + b_0(c_1 + c_2))}{2 b_0^{3/2} c_0^{3/2} b_1}\epsilon^3 + \mathcal{O}(\epsilon^4) \qquad (29)$$

The important point to note is that, compared to the 23 element of Eq.21, the leading order expansion in ϵ has changed. No longer is it at ϵ^3, it is now ϵ^2.

Note that we can write the expressions for the canonically normalized off-diagonal Yukawa matrix elements Y_{23} and Y_{32} in such a way that they would transform into each other if we interchange $b_i \leftrightarrow c_i$, as would be expected. We also note that the diagonal matrix elements would transform into themselves under the same substitution, $b_i \leftrightarrow c_i$. This has been checked explicitly to the order in the Taylor expansion shown in the Appendix.

Setting the $\mathcal{O}(1)$ parameters b_i,c_i and d_i to unity, the Yukawa matrix then takes

the canonical form:

$$Y_c \sim \begin{pmatrix} (a_1+a_2)\epsilon^3 & -a_1\epsilon^2+(1.5a_1+a_2)\epsilon^3 \\ -a_1\epsilon^2+(1.5a_1+a_2)\epsilon^3 & a_1\epsilon-2a_1\epsilon^2+a_2\epsilon^3 \end{pmatrix} + \mathcal{O}(\epsilon^4) \tag{30}$$

We emphasise again that Eq.30 has a different power structure in ϵ to the original, non-canonically normalised Yukawa in eq. (21).

What has happened is that the unitary matrix which redefines our fields has mixed them amongst themselves. This leads to a similar (but different) high energy Yukawa texture. This certainly could be a sufficiently different set-up to ruin any predictions that the non-canonical model was designed to make. However we emphasise that this result applies to the simplified $SU(3)_F$ model with a single expansion parameter, and not the realistic $SU(3)_F$ model of King and Ross [4] with two different expansion parameters.

By comparing the non-canonical Yukawa matrix in eq. (21) to the canonical Yukawa matrix in eq. (30), we can see that the Kähler mixing angles are large, of $\mathcal{O}(\epsilon)$. In the appendix, we have an expression for the inverse P-matrix, P^{-1}. The large mixing effect can come only from the mixing part of the transformation. Schematically, the appearance of the ϵ^2 leading order terms in the off-diagonal elements can then be understood by neglecting all the coefficients of $\mathcal{O}(1)$, as follows:

$$Y_c \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon \end{pmatrix} \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 \\ \epsilon^2 & \epsilon \end{pmatrix} \tag{31}$$

which accounts for the appearance of the ϵ^2 leading order terms in the off-diagonal elements.

3.5 Three generations of matter

The procedure we have discussed for the second and third families can straightforwardly be generalised to include also the first family or indeed to any number of generations. The first thing to do is to write down all of the symmetries of the model.

Having done this, write down all of the non-renormalisable operators up to the the chosen order in the UV cutoff, M. In the two generation case, this was to $\mathcal{O}(M^{-3})$. The next step is to write down the Kähler potential consistent with all the symmetries of the model, up to the same order in the UV cutoff M as the superpotential was expanded to. For tractability, some terms may have to be switched off to make the Kähler metric block diagonal as in eq. (24). At this point, the fields which break the family symmetry are replaced by their vevs.

Then one must find the matrices which canonically normalise each sub-block of the Kähler metric. These will take the form of a unitary matrix which diagonalises the sub-block, and then a rescaling which takes it to the identity matrix of the appropriate size. Having done this, the unnormalised fields can be written in terms of the canonically normalised fields. If $\tilde{P}_S$ is the matrix which diagonalises the sub-block $\tilde{K}_S$, and ψ_S and ψ'_S are respectively the unnormalised and canonically normalised fields in the sub-block, then:

$$\tilde{P}_S \tilde{K}_S \tilde{P}_S^\dagger = \mathbf{1} \tag{32}$$

$$\psi_S = \tilde{P}_S \psi'_S \tag{33}$$

We then substitute eq. (33) into the superpotential. Once we have done this, the canonically normalised Yukawa matrix will be the coefficient of $F'\overline{F}'H'$. At this point, the Yukawa matrix elements may well be of the form of one polynomial in expansion parameters, (ϵ in the example model) divided by another. In this case, to understand the power structure in the expansion parameter, it is necessary to use a Taylor expansion to get a power series in the expansion parameters (we may do this because the expansion parameters must be small in order for the whole technique of non-renormalisable operators to work in the first place).

Having completed this, the end result is canonically normalised three-generation Yukawa matrices, as required. Note that any step of this calculation could in principle

be intractable, and therefore some simplifying assumptions may have to be made.

4 Canonical origin of mixing angles

It is possible in principle that all fermion mixing angles could originate from diagonal Yukawa couplings, via canonical normalisation. To illustrate the idea, consider a two generation model, in which the non-canonical Yukawa is diagonal, with the 33 element dominating over the 22 element:

$$Y_{\text{n.c.}} = \begin{pmatrix} \overline{\epsilon} & 0 \\ 0 & \lambda_t \end{pmatrix} \tag{34}$$

So $\overline{\epsilon} \neq 0$ and $\overline{\epsilon} \ll \lambda_t$ In general, the mixing part of the canonical normalisation can be parameterised by a unitary rotation matrix, U, and the rescaling can be parameterised by a diagonal matrix, R:

$$Y_c = \begin{pmatrix} r_1 & 0 \\ 0 & r_1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \overline{\epsilon} & 0 \\ 0 & \lambda_t \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} \tag{35}$$

This leads to a canonical Y, $\phi = -\theta$

$$Y_c = \begin{pmatrix} r_1^2(\overline{\epsilon}\cos^2_\phi + \lambda_t \sin^2\phi) & r_1 r_2 \frac{\lambda_t - \overline{\epsilon}}{2} \sin 2\phi \\ r_1 r_2 \frac{\lambda_t - \overline{\epsilon}}{2} \sin 2\phi & r_2^2(\overline{\epsilon}\sin^2\phi + \lambda_t \cos^2_\phi) \end{pmatrix} \tag{36}$$

Now consider the values for the parameters that $\sin\phi \approx \epsilon$, $\overline{\epsilon} \approx \epsilon^n$ with $n > 3$, $r_1 \approx r_2 \approx 1$ and $\lambda_t \approx \epsilon$:

$$Y_c \approx \begin{pmatrix} \epsilon^3 + \epsilon^n(1-\epsilon^2) & \epsilon^2(1-\epsilon^{n-1}) \\ \epsilon^2(1-\epsilon^{n-1}) & \epsilon(1-\epsilon^2) + \epsilon^{n+2} \end{pmatrix} \tag{37}$$

By taking the leading order in ϵ and the leading two orders in ϵ in the 33 element, we can get a Yukawa matrix, post canonical-normalisation:

$$Y_c \approx \begin{pmatrix} \epsilon^3 & \epsilon^2 \\ \epsilon^2 & \epsilon - \epsilon^3 \end{pmatrix} \tag{38}$$

This look remarkably like the Yukawa matrix in the full case *before* canonical normalisation, (eq. (21)).

5 Conclusions

We have highlighted the important rôle that canonical normalisation of kinetic terms in flavour models based on family symmetries can play in determining the Yukawa matrices. Even though the kinetic terms may be correctly canonically normalised to begin with, we have shown that they will inevitably be driven into a non-canonical form by a similar operator expansion to that which determines the Yukawa operators. Therefore in models based on family symmetry canonical re-normalisation is mandatory before the physical Yukawa matrices can be extracted.

In SUSY models with family symmetry, the Kähler potential should be considered to the same order in the UV cutoff as one takes in the superpotential. Having done so, the Kähler metric, which follows from the Kähler potential should be canonically normalised. This will save the model from dangerous off-diagonal scalar mass mixing terms in the super-CKM basis (and its leptonic analogue), but the fields appearing in the superpotential must be redefined leading to modified predictions for Yukawa matrices.

We have performed an explicit calculation of such mixing associated with canonical normalisation of the Kähler metric in a supersymmetric model based on $SU(3)$ family symmetry, and shown that such effects can significantly change the form of the Yukawa matrix. In the simplified example considered, one off-diagonal Yukawa element loses one power of an expansion parameter, $\epsilon \approx \frac{1}{10}$, corresponding to that element growing by an order of magnitude. We emphasise that this result does not imply that the full realistic $\mathrm{SU}(3)_F$ model of King and Ross [4] with two different expansion parameters is incorrect. The analysis of the realistic $\mathrm{SU}(3)_F$ model model with two different expansion parameters is more subtle, and such models may remain completely viable after canonical normalisation [8].

We have also pointed out that the canonical form of the scalar mass matrices takes

a universal form as a direct result of the simple supersymmetry breaking mechanism we have assumed. The effects of canonical normalisation on the scalar mass matrices in such realistic $SU(3)_F$ models recently considered in [9] must therefore also be reconsidered [8].

Finally we have pointed out that in principle quark mixing could originate entirely from canonical normalisation, with only diagonal Yukawa couplings before canonical normalisation. Although we have only considered a two family example explicitly, we have indicated how the procedure generalises to the full three family case.

In conclusion, when looking at the flavour problem in effective field theories based on family symmetries, it is not enough just to find operators which gives a viable Yukawa structure. It is also necessary to examine the structure of the kinetic terms, and ensure that the Yukawa structure remains viable after canonically normalising the kinetic terms, which redefines the fields.

Acknowledgements

I.P. thanks PPARC for a studentship. We are also grateful to S.Antusch for useful discussions. S.F.K. would like to thank L.Everett and G.Kane for emphasising the importance of the Kähler potential. We are extremely grateful to Graham Ross for noticing some important errors in an earlier version of this paper. We are also grateful to T.Dent for a useful comment.

A Expressions for the canonically normalised Yukawa elements, and P_{LH}^{-1}

We write here the full expressions for the four Yukawa elements.

$$Y_{22} = a_2 \frac{1}{\sqrt{b_0 c_0 d_4}} \epsilon^3 + a_1 \frac{b_3 c_3}{\sqrt{b_0 c_0} b_1 c_1} \epsilon^3 + \mathcal{O}(\epsilon^4) \tag{39}$$

$$Y_{23} = -a_1 \frac{b_3}{\sqrt{b_0 c_0} b_1}\epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}}\epsilon^3 + a_1 \frac{b_3(b_2 c_0 + b_0(c_1+c_2))}{2 b_0^{3/2} c_0^{3/2} b_1}\epsilon^3 + \mathcal{O}(\epsilon^4) \quad (40)$$

$$Y_{32} = -a_1 \frac{c_3}{\sqrt{b_0 c_0} c_1}\epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}}\epsilon^3 + a_1 \frac{c_3(c_2 b_0 + c_0(b_1+b_2))}{2 b_0^{3/2} c_0^{3/2} c_1 \sqrt{d_4}}\epsilon^3 + \mathcal{O}(\epsilon^4) \quad (41)$$

$$\begin{aligned} Y_{33} = {} & a_1 \frac{1}{\sqrt{b_0 c_0}}\epsilon + -a_1 \frac{c_0(b_1+b_2) + b_0(c_1+c_2)}{2 b_0^{3/2} c_0^{3/2}}\epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}}\epsilon^3 \\ & + a_1 \frac{1}{8 b_0^{5/2} c_0^{5/2}}\Big(\frac{c_0^2(3b_1^4 + 6b_1^3 b_2 - 4b_0^2 b_3^2 + b_1^2(3b_2^2 - 4b_0(b_3+b_4+b_6)))}{b_1^2} \quad (42) \\ & + \frac{b_0^2(3c_1^4 + 6c_1^3 c_2 - 4c_0^2 c_3^2 + c_1^2(3c_2^2 - 4c_0(c_3+c_4+c_6)))}{c_1^2} \\ & + 2 b_0 c_0 (b_1+b_2)(c_1+c_2)\Big)\epsilon^3 + \mathcal{O}(\epsilon^4) \quad (43) \end{aligned}$$

These follow from the expressions for the inverse P-matrix after it has been Taylor expanded in ϵ to order ϵ^3 around the point $\epsilon = 0$. The full expression for the left-handed P-matrix is then, to sub-leading order in ϵ:

$$P_{LH}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{b_0}} - \frac{b_2 \epsilon}{2 b_0^{3/2}} & \frac{b_3 \epsilon}{\sqrt{b_0} b_1} - \frac{(b_1+b_2) b_3 \epsilon^2}{2 b_0^{3/2} b_1} \\ -\frac{b_3 \epsilon}{\sqrt{b_0} b_1} + \frac{b_2 b_3 \epsilon^2}{2 b_0^{3/2} b_1} & \frac{1}{\sqrt{b_0}} - \frac{(b_1+b_2)\epsilon}{2 b_0^{3/2}} \end{pmatrix} \quad (44)$$

The structure of the right-handed equivalent is exactly the same, but with every b_i replaced with a c_i.

References

[1] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147** (1979) 277.

[2] For a reviews of theories fermion masses and further references, see G. G. Ross, "Models of fermion masses," TASI lectures, Boulder, 2000;

H. Fritzsch and Z. z. Xing, Prog. Part. Nucl. Phys. **45** (2000) 1 [arXiv:hep-ph/9912358].

See also S.Raby, Phys.Rev.D66:010001,2002 (page 142)

[3] S. F. King, arXiv:hep-ph/0310204.

[4] S. F. King and G. G. Ross, Phys. Lett. B **574** (2003) 239 [arXiv:hep-ph/0307190].
S. F. King and G. G. Ross, Phys. Lett. B **520** (2001) 243 [arXiv:hep-ph/0108112].

[5] S. K. Soni and H. A. Weldon, Phys. Lett. B **126** (1983) 215.

[6] E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B **369**, 255 (1996) [arXiv:hep-ph/9509410].
E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B **356**, 45 (1995) [arXiv:hep-ph/9504292].

[7] I. Jack, D. R. T. Jones and R. Wild, Phys. Lett. B **580** (2004) 72 [arXiv:hep-ph/0309165].
H. K. Dreiner and M. Thormeier, arXiv:hep-ph/0305270.
H. K. Dreiner, H. Murayama and M. Thormeier, arXiv:hep-ph/0312012.

[8] S.F.King, I.N.R.Peddie, G.G.Ross, L.Velasco-Sevilla and O.Vives, in progress.

[9] G. G. Ross, L. Velasco-Sevilla and O. Vives, arXiv:hep-ph/0401064.